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On MHD boundary-layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux

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Abstract

Purpose – The hydromagnetic mixed convection flow of an incompressible viscous electrically conducting fluid and mass transfer over a vertical porous plate with constant heat flux embedded in a porous medium is investigated.

Design/methodology/approach – Using the Boussinesq and boundary-layer approximations, the fluid equations for momentum, energy balance and concentration governing the problem are formulated. These equations are solved numerically by using the most effective Newton–Raphson shooting method along with fourth-order Runge–Kutta integration algorithm.

Findings – It was found that for positive values of the buoyancy parameters, the skin friction increased with increasing values of both the Eckert number (Ec) and the magnetic field intensity parameter (M) and decreased with increasing values of both the Schmidt number (Sc) and the permeability parameter (K).

Practical implications – A very useful source of information for researchers on the subject of hydromagnetic flow in porous media.

Originality/value – This paper illustrates the effects of magnetic field on mixed convective boundary layer flow past a vertical plate embedded in a saturated porous medium with mass transfer and a constant heat flux.

Keywords Convection, Flow, Porous materials, Magnetohydrodynamics, Heat, Flux

Paper type Research paper

Nomenclature

$\bar{u}, \bar{v}$	Velocity-components	Sh	Sherwood number
g	Gravitational acceleration	u, v	Dimensionless velocity-components
K	Permeability parameter	D	Mass diffusivity
C_p	Specific heat at constant pressure	v_0	Wall suction velocity
$\bar{T}$	Fluid temperature	$\bar{c}_\infty$	Free stream concentration
$\bar{T}_\infty$	Free stream temperature	c_w	Concentration at plate surface
$\bar{q}$	Constant wall heat flux	B_0	Magnetic field strength
G_m	Solutal Grashof number	G_r	Grashof number
M	Magnetic field parameter	U	Free stream velocity
Pr	Prandtl number	Ec	Eckert number

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Sc	Schmidt number	v_0	Constant wall suction
θ	Dimensionless temperature	ϕ	Dimensionless concentration
β_T	Thermal expansion coefficient	σ	Electrical conductivity
β_c	Concentration expansion coefficient	ρ	Fluid density
λ	Thermal conductivity	υ	Fluid kinematic viscosity

1. Introduction

Mixed convection flow with simultaneous heat and mass transfer from different geometrics embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors and underground energy transport. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns (Chandrasekhara *et al.*, 1992). Furthermore, magnetohydrodynamic (MHD) has attracted the attention of a large number of scholars due to its diverse applications (Geindreau and Auriault, 2002). In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere, etc. In engineering it finds its application in MHD pumps, MHD bearings, etc. Workers like Chamkha and Abdul-Rahim Khaled (2000) and Hossain and Mandal (1985) have investigated the effects of magnetic field on natural convection flow past a vertical surface. Mass diffusion effects on natural convection flow past a flat plate were studied by researchers like (Makinde, 2005; Makinde *et al.*, 2003; Martynenko *et al.*, 1984; Muthukumaraswamy *et al.*, 2001). A comprehensive account of the boundary layers flow over a vertical flat plate embedded in a porous medium can be found in Kim and Vafai (1989) and Liao and Pop (2004).

In the present study, we propose to investigate the effects of mass transfer on the free convection flow of a viscous incompressible electrically conducting fluid past a vertical porous flat plate embedded in a porous medium with constant heat flux in the presence of a transversely imposed magnetic field. Solutions are presented in both graphical and tabular form and given in terms of the local skin friction, plate surface temperature and local mass transfer rate for various parametric values. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies (Chamkha and Abdul-Rahim Khaled, 2000; Kim and Vafai, 1989; Martynenko *et al.*, 1984).

2. Mathematical formulation

We consider steady, unidirectional flow of a laminar, incompressible, electrically conducting fluid past a semi-infinite porous vertical plate with constant heat flux embedded in a porous medium in the presence of a transversely imposed magnetic field (see Figure 1). In addition, there is no applied electric field and all of the Hall effects are neglected. Since the magnetic Reynolds number is very small for most fluid used in industrial applications, we assume that the induced magnetic field is negligible. Let the x-axis be taken along the direction of plate and y-axis normal to it, then under the Boussinesq and boundary-layer approximations, the fluid equations for momentum, energy balance and concentration governing the problem under consideration can be written in dimensionless form as (Chandrasekhara *et al.*, 1992; Chamkha and Abdul-Rahim Khaled, 2000; Makinde, 2005).

$$-\frac{du}{dy}=\frac{d^2u}{dy^2}+G_r\theta+G_m\phi+M(U-u)+\frac{U-u}{K}, \tag{1}$$

$$-\frac{d\theta}{dy}=\frac{1}{\Pr}\frac{d^2\theta}{dy^2}+Ec\left(\frac{du}{dy}\right)^2+MEc(U-u)^2, \tag{2}$$

$$-\frac{d\phi}{dy}=\frac{1}{Sc}\frac{d^2\phi}{dy^2}, \tag{3}$$

$$u=0,\ \frac{d\theta}{dy}=-1,\ \phi=1,\quad \text{on}\quad y=0, \tag{4}$$

$$u=U,\ \theta=\phi=0,\quad \text{on}\quad y\to\infty. \tag{5}$$

It is important to note that Equations (1)-(5) are made dimensionless by introducing the following variables and quantities into the governing conservation equations:

$$\begin{aligned} &y=\frac{v_0\bar{y}}{\upsilon},u=\frac{\bar{u}}{v_0},U=\frac{\bar{U}}{v_0},\theta=\frac{\lambda v_0(\bar{T}-\bar{T}_\infty)}{\bar{q}\upsilon},\phi=\frac{\bar{c}-\bar{c}_\infty}{\bar{c}_w-\bar{c}_\infty},G_r=\frac{g\beta_T\bar{q}\upsilon}{\lambda v_0^4},\\ &G_m=\frac{\upsilon g\beta_c(\bar{c}_w-\bar{c}_\infty)}{v_0^3},M=\frac{\sigma B_0^2\upsilon}{\rho v_0^2},K=\frac{v_0^2\bar{K}}{\upsilon^2},Ec=\frac{\lambda v_0^3}{\bar{q}\upsilon C_p},Sc=\frac{\upsilon}{D},\Pr=\frac{\upsilon\rho C_P}{\lambda}.\end{aligned} \tag{6}$$

Other physical quantities of interest in this problem, namely; the skin friction parameter (τ) and the Sherwood number (Sh) can be easily computed. These quantities are defined in dimensionless terms as: $\tau=u'(0)$ and $Sh=-\phi'(0)$, where the prime symbol denotes differentiation with respect to y.

3. Numerical procedure

Equation (3) together with the corresponding boundary conditions Equations (4)-(5) can be solved exactly to obtain

$$\phi(y)=e^{-Scy}. \tag{7}$$

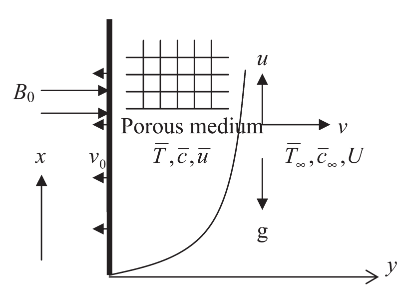

Figure 1.
Flow configuration and coordinate system

However, the set of ordinary differential Equations (1)-(2) with boundary conditions (4)-(5) have been solved by using most effective Newton–Raphson shooting method along with fourth-order Runge–Kutta integration algorithm. Let $u = x_1,\ u' = x_2,\ \theta = x_3,\ \theta' = x_4$. Equations (1)-(2) is then transformed into a system of first order differential equations as follows;

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -x_2 - G_r x_3 - G_m e^{-Scy} - (M + \tfrac{1}{K})(U - x_1) \\ x_3' &= x_4 \\ x_4' &= -\Pr x_4 - \Pr Ec x_2^2 - MEc \Pr (U - x_1)^2 \end{aligned} \tag{8}$$

subject to the following initial conditions

$$x_1(0) = 0, \quad x_2(0) = s_1, \quad x_3(0) = s_2, \quad x_4(0) = -1. \tag{9}$$

In a shooting method, the unspecified initial conditions, s_1 and s_2 are assumed and Equation (8) is then integrated numerically as an initial valued problem to a given terminal point. The accuracy of the assumed missing initial conditions are then checked by comparing the calculated values of the dependent variable at the terminal point with its given value there. If a difference exists, improved values of the missing initial conditions must be obtained and the process is repeated. The computations were done by a written program which uses a symbolic and computational computer language MAPLE. A step size of $\Delta y = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of y_∞ was found to each iteration loop by the assignment statement $y_\infty = y_\infty + \Delta y$. The maximum value of y_∞ to each group of parameters Sc, M, G_r, G_m, K, Pr and Eckert number (Ec) is determined when the values of unknown boundary conditions at $y = 0$ not change to successful loop with error less than 10^{-7}.

4. Results and discussion

In this section, attention is focussed on positive values of the buoyancy parameters $G_r > 0$ (which corresponds to the cooling problem) and $G_m > 0$ (which indicates that

Ec	Sc	M	K	G_r	G_m	$\phi'(0)$	$u'(0)$	$\theta(0)$
0.1	0.10	0.10	1.00	0.10	0.1	–0.10	1.024550769	1.030489230
1.0	0.10	0.10	1.00	0.10	0.1	–0.10	1.045686854	1.314040024
2.0	0.10	0.10	1.00	0.10	0.1	–0.10	1.070545420	1.649133563
0.1	1.00	0.10	1.00	0.10	0.1	–1.00	0.953168370	1.025456973
0.1	10.0	0.10	1.00	0.10	0.1	–10.0	0.902243516	1.023492861
0.1	0.10	5.00	1.00	0.10	0.1	–0.10	1.583222419	1.060664543
0.1	0.10	10.0	1.00	0.10	0.1	–0.10	1.989172433	1.081940325
0.1	0.10	0.10	0.10	0.10	0.1	–0.10	1.922435586	1.049631841
0.1	0.10	0.10	0.01	0.10	0.1	–0.10	5.279949503	1.132546003
0.1	0.10	0.10	1.00	1.00	0.1	–0.10	1.606989205	1.059733632
0.1	0.10	0.10	1.00	1.50	0.1	–0.10	1.955353955	1.083802895
0.1	0.10	0.10	1.00	0.10	5.0	–0.10	4.905791047	1.887809009
0.1	0.10	0.10	1.00	0.10	7.0	–0.10	7.655234857	3.244581757

Table I. Computations showing values of $\phi'(0)$, $u'(0)$ and $\theta(0)$ for $Pr = 1$, $U = 0.5$

the free stream concentration is less than the concentration at the boundary surface). The cooling problem is often encountered in engineering applications; for example in the cooling of electronic components and nuclear reactors. Furthermore, *Pr* is taken to be 1.0 and the values of Sc are chosen in such a way that they represent the diffusing chemical species of most common interest in like H_2, H_2O and Propyl Benzene. The computations have been carried out for various values of the other parameters embedded in the problem as shown in Table I and Figures 1-8.

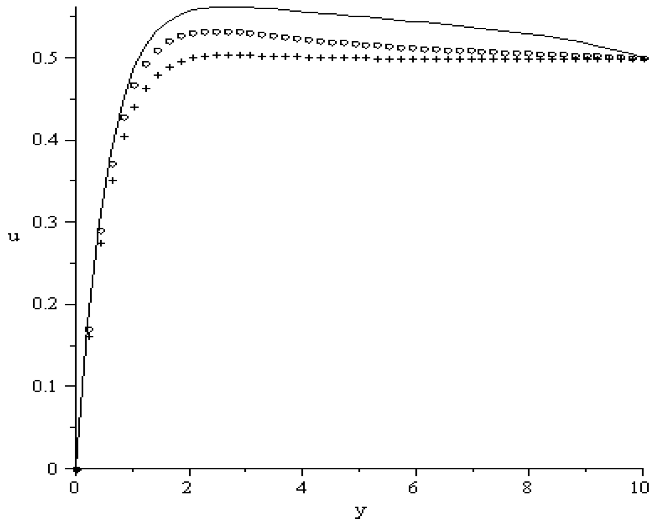

Figure 2.
Velocity profiles
____Sc = 0.1,
oooooSc = 0.3,
++++Sc = 1

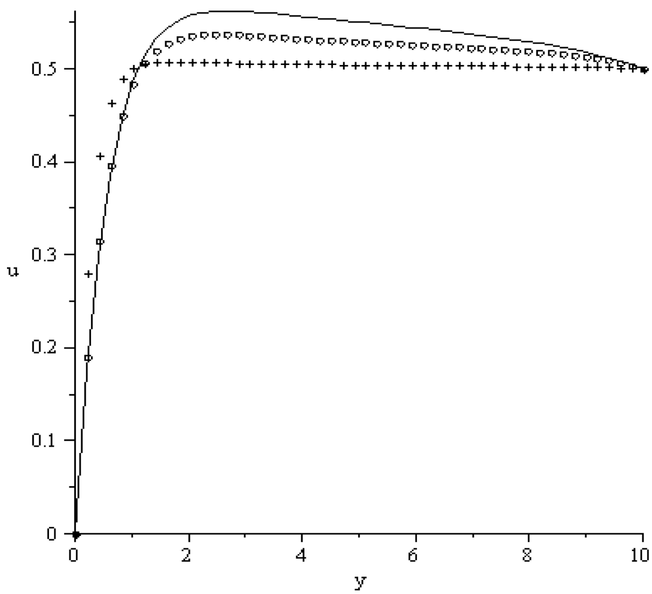

Figure 3.
Velocity profiles
____$M = 0.1$,
ooooo$M = 1.0$,
++++$M = 10$

The results presented in Table I reveals that the skin friction increased with increasing parameter values of Ec, M, G_r, G_m and decreased with increasing parameter values of Schmidt number (Sc) and K. Also, it is interesting to note that the mass transfer rate represented by Sh increases with increasing values of Sc. The velocity profiles are depicted in Figures 2-6 for different values of the important parameters. Generally, the stream-wise velocity profiles show an increase, a peak and then decrease gradually to free stream velocity value far away from the plate. However, it is noteworthy that the

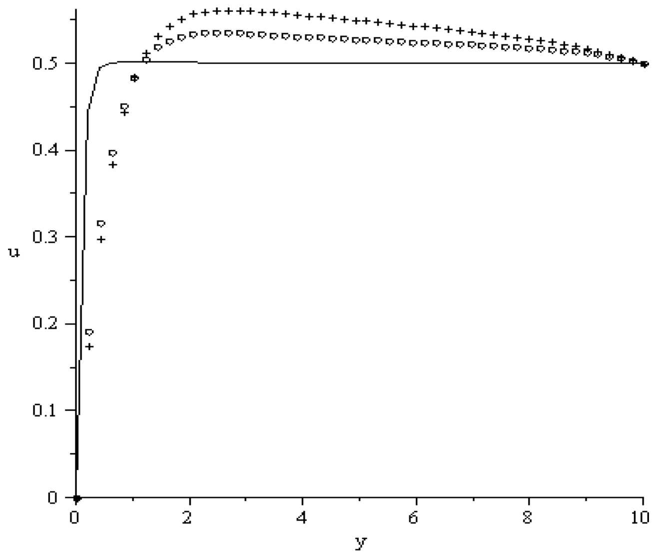

Figure 4.
Velocity profiles
____ $K = 0.01$,
ooooo$K = 0.5$,
++++$K = 1$

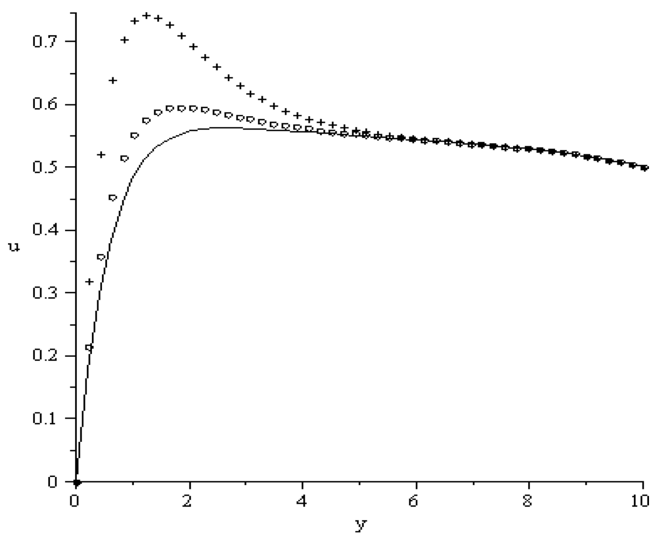

Figure 5.
Velocity profiles
____ $G_r = 0.1$,
ooooo$G_r = 0.5$,
++++$G_r = 1.5$

fluid velocity increases with increasing values of K, G_r, G_m and decreases with increasing values of Sc and M at a small distance away from the plate. The temperature profiles are depicted in Figures 7-8. The fluid temperature attained its maximum value at the plate surface, and then decreases gradually to free stream zero value far away from the plate. It is interesting to note that increasing values of Ec and Solutal Grashof number (G_m) led to a further increase in the fluid temperature profiles. This increase in the temperature profiles are accompanied by simultaneous increase in thermal boundary layer thickness. The fluid concentration profiles as illustrated in Figure 9 reveals an exponential decrease away from the plate. An increase in Sc led to a decrease in the concentration boundary layer thickness.

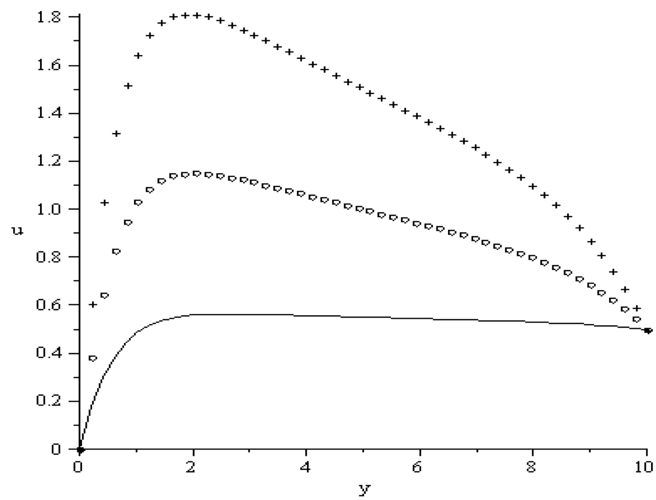

Figure 6.
Velocity profiles
____$G_m = 0.1$,
ooooo$G_m = 1$,
++++$G_m = 2$

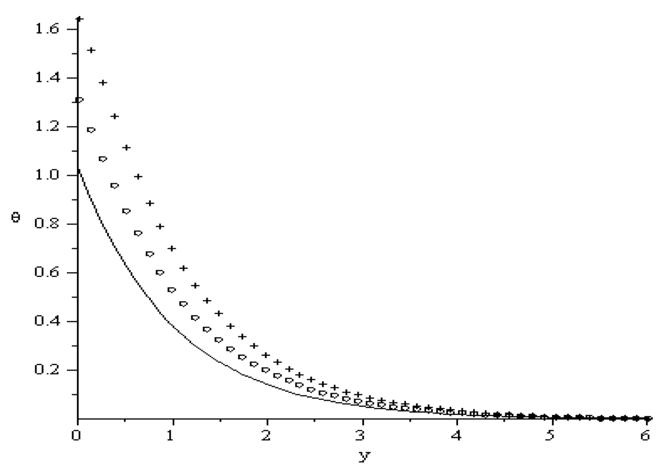

Figure 7.
Temperature profiles
____Ec = 0.1,
ooooo$Ec = 1.0$,
++++Ec = 2.0

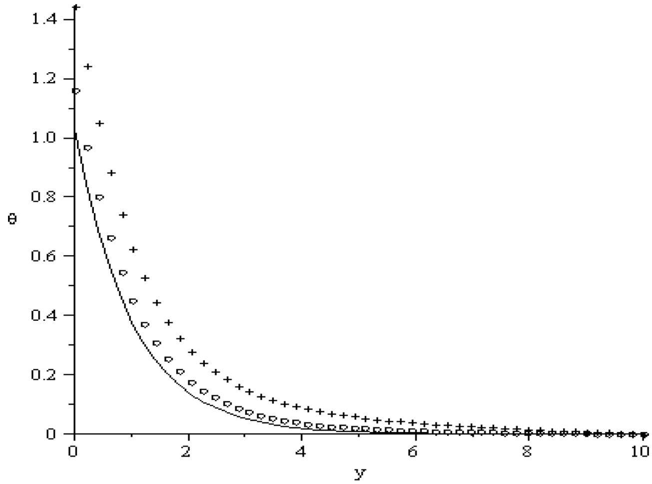

Figure 8.
Temperature profiles
____$G_m = 0.1$,
ooooo$G_m = 1$,
++++$G_m = 2$

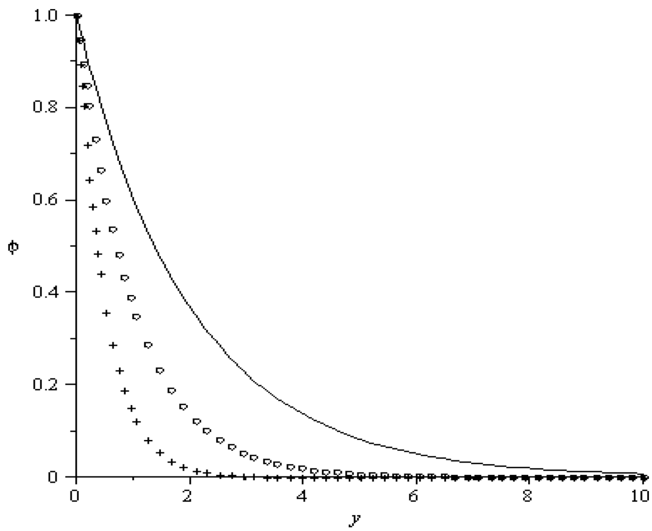

Figure 9.
Concentration profiles
____Sc = 0.5,
oooooSc = 1,
++++Sc = 2

5. Conclusion

Mixed convective boundary layer flow past a vertical porous plate embedded in a saturated porous medium with a constant heat flux and mass transfer in the presence of a magnetic field is investigated numerically. The results reveal among other things that for positive values of the buoyancy parameters, the skin friction increased with increasing values of Ec and magnetic field intensity parameter (M) and decreased with increasing values of Sc and permeability parameter (K).

References

Chandrasekhara, B.C., Radha, N. Kumari, M. (1992), "The effect of surface mass transfer on buoyancy induced flow in a variable porosity medium adjacent to a vertical heated plate", *Heat and Mass Transfer*, Vol. 27 No. 3, pp. 157-66.

Chamkha, A.J., Abdul-Rahim Khaled, A. (2000), "Hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid saturated porous medium", *International Journal of Numerical Methods Heat and Fluid Flow*, Vol. 10 No. 5, pp. 455-76.

Geindreau, C. and Auriault, J.L. (2002), "Magnetohydrodynamic flows in porous media", *Journal of Fluid Mechanics*, Vol. 466, pp. 343-63.

Hossain, M.A. and Mandal, A.C. (1985), "Mass transfer effects on the unsteady hydromagnetic free convection flow past an accelerated vertical porous plate", *Journal of Physics D: Applied Physics*, Vol. 18, pp. 163-9.

Kim, S.J. and Vafai, K. (1989), "Analysis of natural convection about a vertical plateembedded in porous medium", *International Journal of Heat and Mass Transfer*, Vol. 32, pp. 665-77.

Liao, S.J. and Pop, I. (2004), "Explicit analytic solution for similarity boundary-layer equations", *International Journal of Heat and Mass Transfer*, Vol. 47, pp. 75-85.

Makinde, O.D. (2005), "Free-convection flow with thermal radiation and mass transfer past a movingvertical porous plate", *International Communications in Heat and Mass Transfer*, Vol. 32, pp. 1411-19.

Makinde, O.D., Mango, J.M. and Theuri, D.M. (2003), "Unsteady free convection with suction on an accelerating porous plate", *A. M. S. E., Modelling, Measurement & Control*, Vol. 72 Nos. 3-4, pp. 39-46.

Martynenko, O.G., Berezovsky, A.A., Yu. and Sokovishin, A. (1984), "Laminar free convection from a vertical plate", *International Journal of Heat and Mass Transfer*, Vol. 27, pp. 869-81.

Muthukumaraswamy, R., Ganesan, P. and Souldalgekar, V.M. (2001), "The study of the flow past an impulsively started isothermal vertical plate with variable mass diffusion", *Journal of Energy, Heat and Mass Transfer*, Vol. 23, pp. 63-72.

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